

Brussels, 19 May 2025

COST 045/25

DECISION

Subject: Memorandum of Understanding for the implementation of the COST Action “Interactions between Control Theory and Machine Learning” (InterCoML) CA24136

The COST Member Countries will find attached the Memorandum of Understanding for the COST Action Interactions between Control Theory and Machine Learning approved by the Committee of Senior Officials through written procedure on 19 May 2025.

MEMORANDUM OF UNDERSTANDING

For the implementation of a COST Action designated as

COST Action CA24136 INTERACTIONS BETWEEN CONTROL THEORY AND MACHINE LEARNING (InterCoML)

The COST Members through the present Memorandum of Understanding (MoU) wish to undertake joint activities of mutual interest and declare their common intention to participate in the COST Action, referred to above and described in the Technical Annex of this MoU.

The Action will be carried out in accordance with the set of COST Implementation Rules approved by the Committee of Senior Officials (CSO), or any document amending or replacing them.

The main aim and objective of the Action is to boost applications of tools from Control Theory to Machine Learning and vice versa, and to explore the great applicative potential that can be released by combining these two rapidly evolving research areas. This will be achieved through the specific objectives detailed in the Technical Annex.

The present MoU enters into force on the date of the approval of the COST Action by the CSO.

OVERVIEW

Summary

This Action will exploit the deep interconnections between Control Theory (CT) and Machine Learning (ML). It will boost applications of tools from CT to ML and vice versa, and explore the great applicative potential that can be released by combining these two rapidly evolving research areas. In particular, it aims to

- (i) strengthen the control-theoretical foundations of ML methods,
- (ii) leverage modern ML tools to tackle complex and high-dimensional CT problems,
- (iii) develop hybrid and data-driven models for highly complex application scenarios,
- (iv) transform theoretical results into software solutions and practical implementations in industry and society.

Bringing together participants from multiple fields (mathematical analysis, numerical mathematics, control engineering, computer science, data science, etc), the Action will foster interdisciplinary and cross-sector collaboration within a diverse group of experts from academia and industry. It will also combat fragmentation and communication barriers between the ML and CT communities, which often work in parallel on similar problems but using different terminology and tools, and without sufficient communication with each other. This will allow the Action to combine different approaches, creating synergies that will benefit both sides, and leading to progress in both theoretical investigations and applications.

Through the targeted transfer of knowledge and technology to the industrial sector, the Action will bring benefits to broader society. Focus will be given to implementation options in energy systems and personalised medicine, with the aim of improving the sustainability and environmental profile, as well as healthcare outcomes of European citizens.

Areas of Expertise Relevant for the Action <ul style="list-style-type: none"> • Mathematics: Control theory and optimization • Computer and Information Sciences: Machine learning algorithms 	Keywords <ul style="list-style-type: none"> • high dimensional control problems • data driven modelling • neural networks • optimal control • reinforcement learning
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Specific Objectives

To achieve the main objective described in this MoU, the following specific objectives shall be accomplished:

Research Coordination

- To generate datasets for benchmark problems in control that will be used for checking the performance of new ML algorithms.
- To apply novel ML methods to previously intractable CT problems.
- To use CT in order to investigate and enhance the ML algorithms, in particular their efficiency and reliability.
- To develop novel algorithms by exploring and combining ML and CT tools.
- To provide a list of open problems that will be offered to Master and Ph.D. students, as well as to Young Researchers and Innovators participating in the network.
- To apply the new techniques to real-world problems (e.g. power grids, smart houses, digital twins in

healthcare).

- To produce collaborative research results with researchers from different areas.
- To disseminate insights and findings generated within the Action to the research community and the general public.

Capacity Building

- To create a common language between different communities, removing barriers to the communication of knowledge.
- To establish communication channels (seminars, forums, blogging platforms) allowing for a dynamic overview of the progress achieved and the current state of the art in the field.
- To educate the next generation of experts in the fields and to empower talented Young Researchers and Innovators for a successful career in an international environment, by intensive use of Short-Term Scientific Missions (STSMs) and joint educational programs with industrial partners.
- To establish a platform offering industry-related internships to Master and Ph.D. students.
- To improve the gender, geographical and age balance in CT, ML, and related fields, by focusing on young, ITC, and female researchers and innovators.
- To establish a long-lasting successful collaboration between the groups involved.

TECHNICAL ANNEX

1. S&T EXCELLENCE

1.1. SOUNDNESS OF THE CHALLENGE

1.1.1. DESCRIPTION OF THE STATE OF THE ART

Machine Learning (ML) and Control Theory (CT) are two highly interconnected disciplines with numerous links running between them in both directions. Both are dynamic fields of research whose enormous applicative potential has already found its way into our everyday lives. Automatic flight control, the Mars landing, digital twins, and the decarbonisation of the energy system are just a few applications that are changing our world.

Since its emergence as a distinctive scientific discipline within Artificial Intelligence (AI), ML has leaned heavily on established mathematical theories to justify the performance and reliability of the underlying methods and algorithms. These notably include optimisation theory, which underpins the algorithms for training neural networks, and statistics, which provides estimates for the interpolation and extrapolation errors of these methods and algorithms.

As early as last century, ML algorithms were being analysed as dynamical systems [SS97]; and dynamical model representations, whether discrete [H16] or continuous [E17], are now the prevailing standard. This dynamical systems perspective paved the way for the connection of ML and CT, as the latter addresses the behaviour and the optimal configuration of such systems. CT's potential to enhance methods and algorithms in ML as well as to spawn innovations in the field has been broadly recognised [BK19, RZ23], leading to a fertile and rapidly expanding research area. One direction in ML where the control-theoretic approach already plays a central role is reinforcement learning, whose analogies with control policies are well known and have been exploited for decades [S92, B19].

On the other hand, the great progress in CT and its applications, especially in the broad area of control engineering, heavily exploits the integration of new tools from ML. This makes it possible to enrich the classical model-based approach with a data-driven one, and to implement black-box feedback controllers in situations where classical CT does not provide reliable methods [Re19]. The large potential of these emerging paradigms has been reflected in the recent IFAC World Congress 2023 with its vision "Wa: Control for Solving Societal Problems and Creating Societal Values" and in the IEEE CSS Road map: 2030 [A23], which identifies social and technological areas in which CT can have an impact in upcoming years.

Despite all these promising results, the interaction between the data-driven approach in ML and model-based CT is still at an early stage of development [B22]. This opens a vast area for further theoretical investigations and practical improvements, where some of the fundamental open problems and challenges are the following.

1. Better understanding and improving reliability of ML algorithms:

In the context of supervised and unsupervised ML over large datasets, the best performances are typically achieved by complex models that are difficult to interpret, such as deep learning architectures. This creates a deep imbalance between the performance and the theoretical underpinning of such approaches. Although deep neural networks have been shown to be the most powerful models in ML, the underlying reasons for this success are still not completely understood, and there is a growing effort to construct a mathematical theory capable of understanding their inner workings and estimating the reliability of the outputs of the models. Currently, unreliability and lack of transparency are some of the most serious issues in the development and application of ML procedures, and a deep mathematical understanding of the underlying algorithms is essential to address this problem [Be22].

2. Enhancing CT capabilities in high-dimensional settings:

Many control problems, although having a well-established theoretical background, face huge difficulties when it comes to their numerical treatment, especially in high-dimensional settings. To address this issue different versions of model reduction techniques have been developed and explored in recent decades, leading to new insights, and expanding the scope of applications of mathematical modelling. Although such techniques have not yet been fully exploited, further breakthroughs will require the development and use of novel approaches, especially those based on ML tools [GW21].

3. Modelling processes without (complete) information of the underlying models:

Many real-life systems obey complex physical laws or describe processes that are beyond our (full) knowledge, leaving researchers without a complete model – the cornerstone of any analysis. In such situations, the identification of the model and the design of the controller are based on data obtained through experiments and measurements, leading to data-driven modelling. An efficient numerical treatment of such systems should rely on both classical physics-based discretisation methods and ML algorithms, as well as on the interplay between the two. Some impressive results have been achieved in this direction, but they mostly apply to particular technological problems, whereas a systematic and integrated approach is still under development [N21].

To summarise: investigation of interactions between CT and ML is a cutting-edge research priority. ML needs reliable mathematical methods that can explain and guarantee its good performance and generalisation properties; at the same time, future applications of CT depend on novel methods that will allow it to deal with "big data" and exploit the potential of the latter. This state of affairs is the motivation for this Action, which will foster an interdisciplinary and integrated approach to CT and ML by building new bridges and exploiting existing ties between the two fields.

1.1.2. DESCRIPTION OF THE CHALLENGE (MAIN AIM)

This Action aims to boost applications of tools from CT to ML and vice versa, and to explore the great applicative potential that can be released by combining these two rapidly evolving research areas. This can only be achieved by a strong interdisciplinary and cross-sectorial network that will enable share of the knowledge among different groups and fields, which is a crucial requirement for achieving breakthroughs in the field on the level of both theory and applications. A COST Action provides the most suitable framework for establishing such a network.

The main research questions the Action aims to address are based on three distinct but interdependent approaches, each one linked to one of the research Working Groups (WGs) listed in Section 4 of this proposal.

CT for ML

The advent of the control theoretic perspective on learning with deep residual neural networks [E17] has recently been shown to be not only a reliable computational tool [C18], but also a fruitful avenue for providing original theoretical results to a multitude of problems [Be22]. These include adversarial robustness, generative modelling and generalisation bounds, to name a few. Similarly, dynamic Bayesian networks, which are the central paradigm in statistical ML, can be analysed with the help of classical CT tools like Kalman filters. The aim is to explore these connections and contribute to the mathematics of reliable ML.

ML for CT

This approach addresses the application of ML tools, such as kernel methods and deep neural networks to complex models in control theory. These will be combined with traditional control methods, either on the algorithmic level for developing enhanced and provably efficient novel techniques, or for analysis purposes, in order to better understand the opportunities and limitations of ML for the control design. The focus will be on the development of techniques that can handle high-dimensional problems and face the curse of dimensionality.

Hybrid and data driven modelling

The aim is to address control problems built on observed pairs of input-output data. The Action will develop efficient and reliable methods for designing controllers when the underlying model is unidentified or when the underlying laws and processes only allow partial knowledge. While these topics arise in a wide range of potential applications, the Action will focus on control of power systems (solar, wind, smart houses) and construction of digital twins in healthcare and personalised medicine. In connection with industry partners, the Action will explore implementation options that can provide benefits for clean energy and improve healthcare outcomes of European citizens.

The overarching challenge

In addition to answering the particular research questions listed above, one of the main objectives of the Action will be to reduce fragmentation and communication barriers between the ML and CT communities, which are largely disjoint due to their distinctive languages and cultures (cf. [Re19] for a recent effort to unify this gap), as well as to allow for transfer of knowledge and ideas between academia

and industry.

The exponential growth in the number of results in and around ML and CT makes it difficult for experts to stay informed about the current state of the art, even in their own area. It has become almost impossible to keep track of progress when the topic is studied by different research communities, often in parallel and without interaction, resulting in duplication of results and ideas, and a waste of both resources and opportunities. Similarly, many research problems are being addressed from an academic perspective without exploring their potential for application, while for industrial stakeholders it is hard to follow recent scientific advances. These issues can be successfully addressed only through a large international, interdisciplinary, and cross-sectorial network with well-established communication channels that will facilitate efficient and timely transfer of knowledge. The construction of such a network is one of the main goals of this Action.

Relevance and timeliness

The research plan is highly relevant, examining current and emerging paradigms at the cutting edge of control-theoretic approach to ML. It aims to establish new links between CT and ML and to explore interconnections that have not been tackled so far. Finally, it leans on the relevant strategic documents that identify as priority the following areas to be addressed within the Action: interpretable AI, AI for control of technical dynamical systems, AI in medical applications, and AI for renewable energy [E23].

The research plan is timely: although the last few decades have seen significant progress in both CT and ML, which have resulted in many new technological advances, it is unanimously acknowledged that fostering interaction of the two disciplines offers many promising new opportunities [BK19, Re19, A23]. The Action will enable the two communities to exploit these opportunities, and unleash the enormous potential of mathematical tools and methods in the study and improvement of ML procedures. This, in turn, will increase our ability to integrate data for accurate modelling, facilitate handling of problems not treatable by classical methods of applied mathematics, and greatly expand the range of applications.

1.2. PROGRESS BEYOND THE STATE OF THE ART

1.2.1. APPROACH TO THE CHALLENGE AND PROGRESS BEYOND THE STATE OF THE ART

Exploring and exploiting relations between ideas, problems and methods in CT and ML requires integral and interdisciplinary approaches to the challenge. The Action will bring together different communities that are working on closely related topics, but which have often worked in isolation from each other. This will reveal similarities and differences between existing algorithms, improve comparability, and allow for novel developments and applications based on combinations of different approaches and insights. The size and structure of the Network will ensure comprehensive knowledge of current challenges and achievements, as well as access to computer and experimental labs, data sets, and measurements. Joint efforts of model developers, industrial stakeholders, researchers in CT and ML, as proposed here, will result in synergetic effects and offer an excellent opportunity to tackle the challenges described in this proposal.

The research activities will be conducted on three levels:

1. Theoretical (by improving existing and developing novel methods, focusing on complex and high-dimensional problems);
2. Numerical (through development of efficient algorithms, based on existing knowledge, as well as on novel insights that will be obtained during the life of the Action);
3. Practical (by transformation of the results obtained into practical solutions and applications to industrial and other real-world problems).

In this work, the Action will not only employ classical or already well-established tools (Kalman filters, Reproducing Kernel Hilbert Spaces, natural gradient optimisation, model order reduction techniques such as proper orthogonal decomposition, universal approximation theorems, backpropagation, stochastic gradient descent, etc.), but also recently introduced ones. A non-exhaustive list includes: neural ODEs [C18], residual neural networks [E17], mean field analysis of neural networks [M18], transformers [V17], PINNs and other mesh-free tools [Ra19], the Deep-Ritz method [EY18], methods based on the Feynman-Kac formula [Be22], backward stochastic differential equations [H18], time-delay approach [FZ22], explainable reinforcement learning [M23], feature importance ranking, such as SHAP (SHapley Additive exPlanations [LL17]), and DL-ROM (Deep-Learning based Reduced Order Modeling [M24]).

The novelty of the approach is threefold. First, the Action will apply the above tools to problems to which they have not been applied before. Second, it will contribute directly to improving existing methods, enhancing their efficiency and reliability. Finally, it aims to develop novel methods by combining ML and CT techniques.

Progress beyond the state of the art will follow from exploring the research topics listed in Section 4. Specific strategies for addressing the most important ones are presented below.

Obtaining error estimates for ML surrogates

A well-known problem in Machine Learning and particularly in deep learning is the lack of guarantees for the performance of the learned solution. By using the dynamical system perspective of ML algorithms, error estimates from control theory and numerical analysis will be adapted for their use in ML. In particular, the goal is to derive a posteriori error estimates which are rarely available in the context of ML approximations so far. Special attention will be given to physics-informed neural networks (PINNs) that possess the theoretical potential to approximate solutions of differential equations, but for which no a posteriori error control is currently available. The approach will be based on the design of a loss function that is tailored to a specific problem and is based on the differential operator under consideration.

Application of Game Theory to ML

Game theory interacts with ML and CT in many different ways, ranging from generative adversarial networks to ascent-descent dynamics for adversarial robust networks. The former have proved to be very successful at tackling real-life problems [G14], while adversarial robustness is a cornerstone for the safe application of ML [AM18]. A promising research direction is designing ML algorithms through dynamic games for which the theoretical background is fully provided by robust control methods.

Addressing the curse of dimensionality with ML tools

The classical methods of numerical analysis suffer from the so-called curse of dimensionality. By development and application of mesh-free, data-based algorithms (like PINNs) the Action will attack previously intractable problems and obtain a better understanding of the approximation properties of different neural networks architectures (multilayer, residual, convolutional, etc.), not only in terms of accuracy, but also investigating the extent to which they break the curse of dimensionality.

Construction of control functions using ML methods

In [G20, G21] the authors introduced ML-based methods for the construction of Lyapunov functions for dynamical systems. The Action aims to generalise this approach to control systems and to construct control (Lyapunov) functions. These systems will be also analysed from data using the method of balancing [BH17] with the aim of efficient control design and optimal placements of actuators and sensors.

Solving parameterized control problems

Parameter dependent problems form an important building block of many applied control techniques, such as multiobjective optimal control, parameter optimisation problems, or model predictive control. Standard computational methods are often too costly for solving such problems. The aim is to develop efficient and certified algorithms for solving parameterised control problems that exploit and combine reduced order modelling and machine learning methods. Positive results in that direction have been obtained recently [M24, KLM24] opening a vast field for further research and applications. The algorithms will be applied and their efficiency monitored on the aforementioned control scenarios.

Constructive methods for extremum seeking control

Extremum seeking (ES) is a powerful real-time, non-model based, optimisation method for dynamic problems [LK12] that attempts to determine the optimal performance of a control/dynamical system as it operates and does not require full knowledge of the system. The Action's aim is to obtain quantitative and qualitative bounds for ES controls by employing the time-delay approach to averaging theory recently introduced in [FZ22]. The focus will be on non-quadratic and dynamic maps that have not yet been studied using the time-delay approach.

Control of energy systems

The Action will employ data-driven approaches to develop and improve control of complex energy systems. The goal is to obtain better understanding and trustworthy algorithms for operating such systems that can be implemented in smart homes, power grids or smart devices and make energy systems more robust. In particular, the Action will compare ML-based approaches, such as explainable

reinforcement learning [M23] or feature importance ranking [LL17], with classical CT ones (e.g. model-predictive controls), by implementing them on the same system.

Digital twins for personalised medicine

The development of digital twins in healthcare, with patient-specific and predictive models, plays a pivotal role in advancing personalised medicine. A prominent example of this approach is the construction of cardiac digital twins. Yet while they have enormous potential, a current weakness of digital twins is that they are largely limited to specific applications and are not widely applicable across other disciplines [N21]. The Action will develop and blend methods in optimisation, inverse modelling, parameter estimation and sensitivity analysis with ML and deep learning algorithms for their systematic construction.

Although these topics are challenging, positive results are expected due to the carefully chosen and balanced network of experts, who have already made significant contributions in their respective fields. However, the breakthroughs expected from the Action will not come from the follow-up of individual studies, but from the synergistic effects enabled by the network. The integrated and interdisciplinary pursuit of novel theoretical insights, development of efficient algorithms, and innovative applications will serve as an optimal catalyst for pushing the boundaries of understanding in the field.

1.2.2. OBJECTIVES

1.2.2.1. Research Coordination Objectives

The general research objective is to boost the interaction between the areas of CT and ML by fostering interdisciplinary and cross-sectorial collaboration between diverse groups of experts from academia and industry. In fields such as CT and ML the line separating science and engineering is blurred, allowing for a powerful synergy. On the one hand, theory helps to understand and to improve the techniques developed for practical purposes. On the other hand, the applications and techniques suggest new research in mathematics, while the experimental findings shed light on the theoretical results and stimulate the intuition.

The specific research objectives are listed below.

- To generate datasets for benchmark problems in control that will be used for checking the performance of new ML algorithms.
- To apply novel ML methods to previously intractable CT problems.
- To use CT in order to investigate and enhance the ML algorithms, in particular their efficiency and reliability.
- To develop novel algorithms by exploring and combining ML and CT tools.
- To provide a list of open problems that will be offered to Master and Ph.D. students, as well as to Young Researchers and Innovators participating in the network.
- To apply the new techniques to real-world problems (e.g. power grids, smart houses, digital twins in healthcare).
- To produce collaborative research results with researchers from different areas.
- To disseminate insights and findings generated within the Action to the research community and the general public.

1.2.2.2. Capacity-building Objectives

The overall objective of capacity-building is to strengthen European scientific capacities by bringing together leading groups from various communities targeting similar kinds of problems (mathematicians, computer scientists, control engineers, etc.). In particular, the aim is to create a diverse and dynamic environment of experienced and young researchers that will foster long-lasting, international, interdisciplinary and cross-sectorial cooperation on CT, ML, and neighbouring fields. Only such an approach, together with the resulting synergy effects, allows for novel breakthroughs in these rapidly evolving research fields. The COST Action provides the ideal framework for building the necessary network.

The specific capacity-building objectives are as follows.

- To create a common language between different communities, removing barriers to the communication of knowledge.
- To establish communication channels (seminars, forums, blogging platforms) allowing for a dynamic overview of the progress achieved and the current state of the art in the field.

- To educate the next generation of experts in the fields and to empower talented Young Researchers and Innovators for a successful career in an international environment, by intensive use of Short-Term Scientific Missions (STSMs) and joint educational programs with industrial partners.
- To establish a platform offering industry-related internships to Master and Ph.D. students.
- To improve the gender, geographical and age balance in CT, ML, and related fields, by focusing on young, ITC, and female researchers and innovators.
- To establish a long-lasting successful collaboration between the groups involved. In particular, the Action will be used as a platform for preparing and submitting proposals for EU or national grants to fund research activities that are not COST eligible, as well as to extend the cooperation beyond the timeframe of the Action.

2. NETWORKING EXCELLENCE

2.1. ADDED VALUE OF NETWORKING IN S&T EXCELLENCE

2.1.1. ADDED VALUE IN RELATION TO EXISTING EFFORTS AT EUROPEAN AND/OR INTERNATIONAL LEVEL

As ML becomes omnipresent in everyday life, there are a great number of projects dealing with applications of ML in various fields, many of which are underway, or have been completed, within the European Research Area; however, far fewer projects and networks specialise in the broad field of Scientific ML and in advancing the mathematical understanding of ML. Consequently, theoretical progress in the field lags several steps behind the impressive achievements in applications and practice.

Several recently launched national initiatives are trying to fill this gap by supporting not only applications but also research in the field of ML and more general AI, in particular:

- The Priority Program "Theoretical Foundations of Deep Learning" of the German Research Foundation.
- FAIR (Future Artificial Intelligence Research), the Italian foundation supporting research in the Artificial Intelligence sector.
- COPI2A, The Spanish Network on Control and Inverse Problems of systems governed by ODEs and PDEs and Machine Learning.
- The French National AI Research Program (PNRIA).

These projects are mostly national and they do not provide the networking opportunities necessary for creating the synergizing effects that the proposed Action is aiming for. Furthermore, most of the associated networks are rather small in size or have a narrow application focus, e.g., on solving inverse problems or partial differential equations.

Also, it seems that there is a lack of such initiatives at the European level; some isolated examples are the ERC-funded ongoing projects "Control for Deep and Federated Learning" (ERC-2022-ADG) and "Deep Learning: 2.0" (ERC-2021-CDG), as well as the pan-European AI network of excellence ELLIS (European Laboratory for Learning and Intelligent Systems). These projects, however, either lack the funding for extensive networking activities or a substantial mathematical and theoretical focus.

Also at the level of COST Actions, there are emerging opportunities to increase the representation of mathematics and the foundations of ML. A few COST Actions which focus on the applications of ML on specific fields are:

- CA18131 (Statistical and ML techniques in human microbiome studies),
- CA17137 (A network for Gravitational Waves, Geophysics and ML),
- CA21160 (Non-globular proteins in the era of ML).

but they do not deal with the analysis of ML from a mathematical, or even theoretical, point of view.

Some Actions which have a distinct mathematical focus but do not directly touch upon ML:

- CA22137 (Randomised Optimisation Algorithms Research Network),
- CA21169 (Information, Coding, and Biological Function: the Dynamics of Life),
- CA18232 (Mathematical models for interacting dynamics on networks).

The proposed Action aims to fill this gap by fostering integrated and interdisciplinary study, development, and application of ML and CT tools. It will be the first Europe-wide initiative to tackle the mathematical underpinnings of ML. It will encourage dissemination of new ideas and will pursue unified conceptual

approaches to different problems.

The proposed Network will bring together research groups from across the European Research Area, and create, and at the same time benefit from, synergetic effects from many national and international projects. It will build a platform for interactions between theoreticians and software engineers, and leverage and complement existing projects and structures in order to contribute to the next stage in the evolution of ML: the development of explainable ML algorithms.

In collaboration with industry partners (including SMEs as well as large companies), the Action will foster the transformation of theoretical advances into new technologies. Obviously, transfer of mathematical results through numerical algorithms into software solutions and practical applications requires the collective efforts of researchers coming from diverse communities and with different specialisations. The proposed Network possesses the capacity for this challenge and it will strongly support transectorial cooperation with the final aim of maximising its impact and bringing benefits to broader society.

Finally, the Action will be also used as a platform for the preparation and submission of proposals for EU or national grants. This will allow funding of research activities that are not COST eligible (employment of young researchers, Ph.D. tuitions, purchase of equipment, conduct of experiments, etc.), as well as the extension of the cooperation beyond the timeframe of the Action. Consequently, the Action will have a long-term impact on the European Research Area and strengthen its competitiveness with the world's most advanced regions in this rapidly developing field of science and technology.

2.2. ADDED VALUE OF NETWORKING IN IMPACT

2.2.1. SECURING THE CRITICAL MASS, EXPERTISE AND GEOGRAPHICAL BALANCE WITHIN THE COST MEMBERS AND BEYOND

The initial network proposing this COST Action featured a critical mass of experts with proven track records in their respective fields, along with talented Young Researchers and Innovators. Almost all participants have experience in implementing national or international projects which intersect with some of the topics the proposed Action will investigate. In addition, given that CT has always been a research topic at the intersection of many fields, the initial group is multidisciplinary. Most of its members are primarily mathematicians, with an excellent balance between theory, applications and numerical methods. The rest were proposers with primary expertise in computer science, ML and engineering. In the case that as the Action develops more expertise in some of the above areas or related fields should be required, the Network members will utilise their contacts and recruit suitable participants in order to grow and adjust the network.

A particular strength of the Action will be the active involvement of business enterprises dealing with real-life problems. These companies range from innovative AI startups, over research-oriented medium-scale enterprises, all the way up to global players. Individually, both ML and CT have a strong record of knowledge transfer from science to practice. Together, these pathways will allow the transfer of novel insights and developments on the ML-CT combination to industry, businesses and applied research labs, as well. Industrial stakeholders will benefit from collaborations with academic research groups, while, in turn, the investigations of the Action will be motivated and directed by real-world problems.

The initial proposers were distributed among 13 COST Full Members countries (7 of them being ITC), and one COST Cooperating Member. The proposers have strong and diverse backgrounds, with extensive research experience and contacts across Europe and worldwide. The Action will exploit and enhance personal and institutional relations established through previous ad hoc joint work in CT and ML.

Trans-European geographical diversity will be pursued in all Action events. These will be organised in different regions, with an emphasis on ITCs. In this way the Action aims to overcome the fragmentation of research capacities and strengthen networking among different regions within Europe, and beyond.

It is expected that the Action will expand significantly over time, bringing in new research groups and thus unifying a significant portion of European research potential in the field. Here various outreach activities will be employed, such as showcasing success stories of ML-CT application in industrial problems at international conferences and workshops, and promoting the Action through its seminar and events. The ultimate goal is to create a vibrant network capable of facing the challenge of bridging the gap between Scientific and Practical ML via CT approaches.

2.2.2. INVOLVEMENT OF STAKEHOLDERS

The most relevant stakeholders are universities, research institutes, and private enterprises working on practical ML projects. These institutions play a pivotal role in the development and application of ML solutions, ensuring a firm connection between theoretical advancements and practical implementations. Stakeholders will be involved through traditional activities, such as annual meetings, workshops, etc., as well as through more non-conventional events like team-building with outdoor activities or sports. The traditional methods of engagement will provide a platform to share knowledge, insights, and progress updates, fostering collaborative efforts and ensuring a comprehensive understanding of the challenges and opportunities within the ML domain. Team-building events will be integrated into or combined with scientific meetings and play an important role in fostering a creative atmosphere among the members of the Action, integrating Young Researchers and Innovators, and strengthening the common spirit.

Besides the regular in-person meetings which are indispensable to keep the main stakeholders updated on scientific progress and to foster a sense of community, the Action also foresees an important role for regular online seminars. These will reduce the ecological footprint of the Action, and will enhance accessibility and participation, especially for stakeholders from developing countries who may not have the resources to participate in person, thus promoting wider engagement and knowledge dissemination in the fields of ML and CT. Through the seminars, the Action will identify highly engaged and proactive groups, as well as promising research pathways. The Action will support such groups, and facilitate them to access necessary tools, data, and expertise crucial for the successful execution of their projects, ultimately enhancing the overall progress and impact of the initiative.

Young Researchers and Innovators will be involved at all levels of the Action. Care will be taken to ensure their representation in the Action leadership and involvement in the (co-)organisation of scientific events. They will have the opportunity to participate in seasonal schools with the participation of representatives from SMEs and to work on open problems under the supervision of senior experts in the field. Industry-related internships will be offered to Master and Ph.D. students, bridging the gap between academic learning and real-world application. These initiatives will enable Young Researchers and Innovators to gain valuable hands-on experience within relevant industries, fostering a more seamless transition into the professional sphere.

By working closely with industry partners, the Action can tap into practical insights and expertise, aligning academic research with real-world needs and boosting the relevance and impact of its work. To leverage the existing partnership and attract more industry collaborators, the Action will also work in tandem with professional companies specialised in bridging the gap between industry and academia. The transectorial collaboration will be additionally fostered through organisation of on-site events linking researchers, software developments and companies. Networking with enterprises will also facilitate connections with potential end-users of the generated ML solutions, including healthcare institutions and communication companies, among others, thus ensuring the relevance and applicability of the research outcomes in addressing real-world challenges and needs.

The engagement of stakeholders will also be enhanced by launching calls for STSMs and encouraging participation at conferences, especially for Young and Women Researchers. This will promote knowledge exchange and collaboration among researchers, and internationally advertise the Action. Upcoming activities will be announced on the Action website, on the websites of participating universities and institutions, and will be shared through institutional press and social media releases. The Action will use these and similar channels to increase its visibility, attract the attention of a broader audience and gradually involve new stakeholders, including researchers and industry professionals in the network. This approach will contribute to the expansion of the network and the cultivation of a diverse and engaged community committed to advancing ML research and applications.

3. IMPACT

3.1. IMPACT TO SCIENCE, SOCIETY AND COMPETITIVENESS, AND POTENTIAL FOR INNOVATION/BREAKTHROUGHS

3.1.1. SCIENTIFIC, TECHNOLOGICAL, AND/OR SOCIOECONOMIC IMPACTS (INCLUDING POTENTIAL INNOVATIONS AND/OR BREAKTHROUGHS)

ML is radically transforming our approaches to technological and scientific problems, across all the areas of Science and Engineering, including the domain of CT. However, unlike many other fields, CT offers something in return: valuable tools for improving understanding and design of ML algorithms. For example, the construction of learning systems always involves optimisation processes, a core area of CT expertise. This Action strongly advocates for the integration of ML and CT, as their synergies promise to accelerate breakthroughs that neither field could achieve independently.

This COST Action will drive substantial scientific and technological progress by bringing together some of Europe's leading researchers in CT and ML. It will establish a robust network of experts from both academia and industry, dedicated to harnessing the untapped potential at the intersection of these domains.

By advancing our understanding of ML algorithms, this initiative will pave the way for improved methods, with far-reaching impacts across multiple sectors. In particular, the Action will address how CT can make ML algorithms more reliable (e.g., T1.1, T1.2), robust (e.g., T1.3), and explainable (e.g., T1.4). (T refers to specific research topics, listed in Section 4.) Efficiency of the numerical methods will be improved by combining traditional tools with the ML ones (e.g., T2.1, T2.2), particularly in the areas of control systems and optimal control.

The development of new approaches and contributions to explainable ML will represent great added value for the industrial partners. It will empower them to successfully deal with obstacles often encountered in empirical optimisation in industrial development, which may be eliminated thanks to better theoretical preparation of the architecture, model, and training processes. Participation in the Action's activities will give non-academic members an insight into current research advances in ML and CT and facilitate the design of improved software and industrial solutions.

Furthermore, CT plays a vital role in addressing multiple large-scale challenges confronting European society. The recent IEEE CSS Roadmap "Control for Societal-scale Challenges: Road Map 2030" [A23] identifies various topics where this becomes particularly obvious and this COST Action addresses a number of them, in particular those related to sustainability and future energy systems as well as precision medicine and healthcare. These include:

- The hybrid optimal control techniques to be developed in T3.3 can be applied to modern combined gas and steam power plants, as well as to transmission system operators, increasing their efficiency and making future energy systems more sustainable and robust.
- The controllers to be developed in T2.4 will help to prolong the life cycles of materials, thus increasing the sustainability of technical components.
- Soft robotics and medical applications are groundbreaking fields that can benefit hugely from progress in the approximation of the control-to-state mapping as addressed in T3.1. Particularly important is the development of data-driven algorithms capable of providing engineers and doctors with quick responses to stimuli in the specific systems they need to analyse.
- Research in T3.4 will enhance healthcare for EU citizens by applying precision and computational medicine driven by advanced ML algorithms. This will lead to faster and more accurate diagnostics and prognostics, ultimately improving patient care and reducing healthcare costs. Additionally, the development of digital twins in healthcare will transform personalised medicine, allowing for tailored treatments and therapies.

By fostering collaboration between academic researchers and industry partners, the project will accelerate the translation of cutting-edge research into real-world solutions. These advancements will empower companies to innovate in fields such as autonomous systems, robotics, energy optimisation, smart manufacturing, and healthcare. Ultimately, the project's outcomes will lead to societal benefits, improving technological solutions in everyday life, from smarter infrastructure to safer, more efficient

systems. These innovations will not only boost industrial competitiveness but also enhance public services, contributing to societal well-being and resilience in the face of complex global challenges.

In addition, the resulting network of researchers will facilitate the preparation and submission of interdisciplinary EU projects and will make Europe internationally competitive in ML research for complex engineering problems. Special attention will be given to funding options supporting collaborations between scientists and companies. This will create additional opportunities for Action members, especially for Young Researchers and Innovators, Women Researchers, and researchers from ITCs.

In summary, the COST Action will significantly advance the development of innovative methods to tackle pressing societal challenges through groundbreaking scientific and technological innovations. By leveraging advanced algorithms at the intersection of CT and ML, the Action will create powerful solutions that can effectively address critical issues facing our society today.

3.2. MEASURES TO MAXIMISE IMPACT

3.2.1. KNOWLEDGE CREATION, TRANSFER OF KNOWLEDGE AND CAREER DEVELOPMENT

The advent of ML as a driver of innovation in various scientific and technological fields is one of the most striking developments in the 21st Century. The Action aims to tap into this development by exploiting the transformative potential of synergies between CT and ML and by forging innovative and previously unexplored connections. It will also facilitate the translation of insights from academia to industry and vice versa, effectively communicating societal and industrial needs to stimulate researchers.

The Action will create a framework for establishing long-term collaborations of researchers from the ML and CT communities together with those from other disciplines as well as industrial partners, interested in applications of ML and CT. The collaboration of this unique union of domain experts in the Action provides a fertile ground for generating novel theoretical results, efficient algorithms, and innovative applications. By pursuing these ambitious goals, the Action will serve as a catalyst for pushing the knowledge frontiers in the domain.

To ensure efficient knowledge transfer within this network and beyond, the Action will organise special sessions at top-level conferences, organise thematic seminars, and conduct Workshops and Training Schools. For a more rapid and direct transfer of knowledge the Action will also actively engage with a wider public using social media, and design and maintain an Action website which will be regularly updated with recent developments and open problems.

Special attention will be given to transectorial transfer, i.e. to knowledge transfer between the academic and the private sectors. Through the Action's tools and activities its industrial partners will be kept informed of the most recent developments in the field, while, in turn, they will provide the academic researchers with real-world problems on which their research activities can be focused. Transectorial collaboration will be additionally fostered through organisation of on-site events linking researchers, software developments and companies. These measures will ensure that advances in research topics also result in practical solutions and real-life applications that will have an impact on a wider audience.

The Action activities will provide multiple opportunities for the career development of Young Researchers and Innovators. In particular, the above-mentioned training schools will be specifically directed to Young Researchers and Innovators, rapidly taking them to the research frontiers of ML. With the help of STSMs the Action will also help them to kickstart their scientific careers. A special focus of the Action will be to support mentoring relations within the network that will allow Young Researchers and Innovators to work on open problems under the supervision of senior experts in the field. In a similar spirit, interaction with industrial stakeholders will create opportunities for Young Researchers and Innovators to enter the labour market as well as for companies to hire skilled employees. The Action will also support industrial internships of Master or Ph.D. students, allowing them to transfer their academic knowledge to the private sector and, in turn, to be trained in the use and development of state-of-the-art algorithms and cutting-edge applications such as, for instance, the control of autonomous vehicles. This transfer will also be supported by cross-sectorial STSMs involving industry hosts and academic participants, and vice-versa.

Special attention will be focused on enhancing career development for young Women Researchers, recognising the crucial role of gender balance in fostering innovation and socioeconomic growth. To promote their involvement, leadership positions within the Action will be prioritised for female

participants. Additionally, the Action will amplify the visibility of women as role models by actively promoting their research on the Action website. This commitment not only supports individual careers but also contributes to a more diverse and equitable research environment, ultimately benefiting society as a whole.

3.2.2. PLAN FOR DISSEMINATION AND/OR EXPLOITATION AND DIALOGUE WITH THE GENERAL PUBLIC OR POLICY

Throughout the Action's lifetime, the Action will prioritise targeted dissemination activities to effectively share knowledge and insights within academic and industry communities, as well as with the general public. This comprehensive strategy aims to enhance awareness of the initiative's progress and outcomes, using various channels and methods tailored to different audiences for maximum outreach. The Action will leverage social media, workshops, conferences, and publications to facilitate ongoing dialogue and feedback, ensuring that the information reaches those who can benefit from it most and fostering a culture of collaboration and innovation.

The main communication platform for researchers and industrial partners actively involved in the Action will be an online platform, with blogging and discussion components, hosting both internal and external web pages. This platform will also serve as a tool for reaching out to a wider audience. The blogs will be in a concise, informal and engaging style, but at the same time clear and precise.

For the research community at large, the results and ideas will be disseminated through scientific articles in high-level journals, talks and special sessions at prestigious conferences as well as through workshops open for participation of researchers outside of the Action. The planned review papers will be used to promote the research conducted at the level of WGs and the whole Action, as well as new exciting developments in the research areas of CT and ML.

The discussion forum and organisation of online seminars will help to keep the Action's members and relevant stakeholders engaged throughout the course of the Action and also extend its geographical reach. These tools will facilitate collaboration with researchers based outside COST Member Countries, for whom travelling to Europe can pose significant financial and other challenges.

The grantees of STSMs will be required to produce reports describing the research conducted during their STSMs and highlighting its impact and relevance. These reports will be published on the Action's online platform, in this way spreading the knowledge gained among the Network. The Dissemination Conference Grants will be used to support the presentation of Action results at top-level interdisciplinary conferences both within and outside Europe, in order to reach researchers in other disciplines and other regions, as well as partners in industry. These events will also be used to present success stories of ML-CT application to industrial problems.

The final Action conference will also welcome non-Action members. This inclusive approach encourages broader participation and collaboration, fostering an enriching environment for knowledge exchange and networking. The proceedings of the final Action conference will give an overview of the most effective research conducted over the course of the Action. It will serve as a vital platform for disseminating the Action's research outcomes well beyond its conclusion, ensuring ongoing engagement and knowledge sharing with a wider audience.

Finally, dissemination actions and initiatives will be undertaken to guarantee the spread of results to the general public and increase its awareness in the importance of fundamental research for sustainable paradigms in industry. The Action will use social networks to inform the general public about the results obtained and events organised by the Action. The Action will also be advertised through local and European media, as well as through public lectures. To raise public awareness of the significance of CT and ML in everyday life, the Action will actively engage in various popular dissemination activities at local, national, and international levels. This includes public lectures, open-door events, European Researchers' Night, science festivals, and interactive workshops. These initiatives aim to demystify CT and ML concepts, showcasing their real-world applications and benefits. By fostering an understanding of these technologies, the COST Action aims to inspire curiosity and encourage dialogue between researchers and the community.

Leveraging all the aforementioned measures, the Action aims to achieve its long-term goal: to establish a sustainable and diverse network capable of addressing the numerous challenges that lie ahead in the

synergistic fields of CT and ML.

4. IMPLEMENTATION

4.1. COHERENCE AND EFFECTIVENESS OF THE WORK PLAN

4.1.1. DESCRIPTION OF WORKING GROUPS, TASKS AND ACTIVITIES

The Action will be organised into five Working Groups (WGs): three research WGs, one WG dedicated to transformation of theoretical results into practical solutions (WG4), and one dedicated to dissemination and outreach (WG5). Each WG will be coordinated by its Leader and a Vice-Leader, elected at the initial Management Committee (MC) meeting. WG (Vice-)Leaders coordinate and manage activities of the WG and are responsible for the completion of specific deliverables.

Each research WG1-WG3 corresponds to one research question described in Section 1.1.2. Besides addressing specific research topics listed below, each of them will have additional common tasks, related to organisation of networking activities and administration of the WG and the Action.

WG4 will be responsible for transforming theoretical results into software solutions and practical implementations in industry and society. Its activities are essential for bridging the gap between theory and real-world applications, ultimately ensuring the relevance and accessibility of the findings to the broader community. Special attention will be given the following activities: i) creating an open-source repository hosting solutions related to the theoretical advances from WG1-WG3; ii) development of comprehensive documentation and training materials (tutorials, case studies, etc); iii) establishing partnership with industry to promote the adoption of these solutions and validate their applicability.

The aim of WG5 is to provide communication channels that will allow for efficient and timely transfer of knowledge among Action members and towards a wider audience. In particular, to produce a dynamic overview of the progress achieved and the current state of the art (the results, developed codes, open problems, industrial perspectives) that will be immediately available to the other WGs, and later to all the interested researchers. Special attention will be paid to the problems of fragmentation of communities interested in CT, ML and their applications. This WG will help to tackle the challenge that the same or closely related problems/methods are known by different communities under different names, and cross-references are not clearly established.

The WGs will not work in isolation, but rather interactions will be systematically fostered, and the COST Action looks forward to synergistic and spillover effects. Each WG will report its activities to the MC (at least one report per MC meeting), which will coordinate their cooperation. Communication within WGs will be based on regular weekly email correspondence and remote web-conferences whenever needed.

The specific tasks and activities of the WGs are as follows.

WG1. CT for ML

- T1.1 Obtaining error estimates for ML surrogates
- T1.2 Theoretical guarantees for physics-informed neural networks
- T1.3 Application of Game Theory to ML
- T1.4 Kalman filters in statistical ML
- T1.5 Analysis of high-dimensional deep neural networks and transformers using CT tools

WG2. ML for CT

- T2.1 Addressing the curse of dimensionality with ML tools
- T2.2 Solving parameterised optimal control problems
- T2.3 Construction of control Lyapunov functions using ML methods
- T2.4 Developing ML-based approaches for the for the life-cycle-optimisation in materials
- T2.5 Exploiting PINNs for solving complex free boundary problems

WG3. Hybrid and Data-driven modelling

- T3.1 Approximating the (control to) state law from data
- T3.2 Constructive methods for extremum seeking control
- T3.3 Control of energy systems
- T3.4 Digital twins for personalised medicine

In addition to addressing the above research topics, each of the research WG1-WG3 will have the following common tasks.

1. Organise Training Schools and Workshops (at least two events per year for the whole Action).
2. Organise sessions and/or mini-symposia within specialised conferences organised in Europe (e.g. ECC, ECML-PKDD, IJCAI-ECAL-26).
3. Prepare and submit joint proposals for EU or national grant agencies to fund research activities that are not COST-eligible.
4. Prepare survey articles on the topic at the end of the Action.
5. Prepare a final report for each WG in the form of a white paper.

WG4 Transformational activities

1. Identify possible real-world applications of the results gained in WG1-WG3 in collaboration with industrial partners.
2. Establish a public repository for data, coding and programs (on GitHub or a similar platform) in accordance with the FAIR data principles.
3. Develop comprehensive documentation and training materials (tutorials, case studies, etc).
4. Identify and pursue funding opportunities that facilitate collaborations between scientists and industry stakeholders.
5. Organise on-site events linking companies, software developers and researchers (at least two such events per the Action lifetime).

WG5. Sharing & spreading

1. Develop an online platform for sharing the results and ideas of the researchers involved in the Action, both within and beyond the Action.
2. Provide a forum for discussion and communication to be used by all Action members.
3. Launch and run on a regular basis an (online) seminar of the Action.
4. Provide a list of open problems that will be offered to Master and Ph.D. students, and to Young Researchers and Innovators participating in the network.
5. Establish a platform offering industry related internships to Master and Ph.D. students.
6. Disseminate results via social media.
7. Organise participation in popular science activities at local, national and international level (public lectures, Open Doors events, European Researchers Night, Science festivals, etc.).

The MC will oversee the progress of the Action, the allocation and use of funds, and make strategic decisions concerning the management of the Action, meeting annually. The MC will delegate its powers related to continuous, non-strategic management of Action activities to the Core Group.

The Action's activities will be coordinated by the Core Group composed of the Action Chair (AC) and Vice-Chair, WG Leaders and Vice-Leaders, Science Communication Coordinator (SCC), Grant Awarding Coordinator (GAC), and the Equal Opportunities Coordinator (EOC), all to be elected in the first MC meeting, as well as the Grant Holder Scientific Representative (GHSR). The Core Group will maintain regular contact and will meet, in person or via videoconference, whenever needed.

The EOC will be in charge of ensuring gender, geographical and age balance in all Action activities. In particular the EOC will be in charge of preparing annual progress reports including gender, geographical and age distribution of participants in the Action activities, as well as suggestions for improving representation and outcomes where necessary.

The roles of SCC and GAC, as well as of other mandatory leadership positions, are in accordance with the Annotated Rules for COST Actions (Article 3.9).

4.1.2. DESCRIPTION OF DELIVERABLES AND TIMEFRAME

The following list presents the Action deliverables and timeframe (Y: year, Q: Quarter).

1. Science communication plan (Y1 Q2).
2. Action webpage which will also include a blogging platform for sharing the Action activities and results (Y1 Q2).
3. Online forum for discussion and communication (Y1 Q2).
4. A(n online) seminar of the Action (Y1 Q2).
5. An open-source software repository (on GitHub or a similar platform) (Y1 Q2).
6. A list of open problems offered to Young Researchers and Innovators included in the network, to be maintained throughout the Action, and beyond (Y1 Q3).
7. A platform offering industry related internships to Master and Ph.D. students (Y1 Q3).
8. A library of possible applications, suggested by industrial partners and other participants, to be maintained throughout the Action, and beyond (Y1 Q4).
9. Book of abstracts for each workshop/conference organised within the Action (continuously).
10. One page summary report following each STSM (continuously).
11. Final reports published as a white paper for each WG 1-4 (Y4 Q4).
12. A collection of survey articles on the topic published by a prestigious publisher after the Final Action Conference (Y4 Q4).

4.1.3. RISK ANALYSIS AND CONTINGENCY PLANS

The general risk is that some of Action objectives (specified in Section 1.2.2) will not be fully accomplished. To prevent such scenarios, their realisation will be continuously monitored and analysed in the annual progress reports. The objectives are formulated according to the SMART guidelines, which make it possible to measure/quantify the level of their realisation, as well as the overall Action output, and to detect if the progress is insufficient. In the latter case, the Core Group will propose solutions and entrust the relevant WG leader(s) who will, in turn, implement appropriate measures depending on a specific objective.

The particular risks and the corresponding measures to minimise them are in order.

- **Lack of progress in a specific topic.** As the proposed research is based upon new ideas and implementation of novel methods at the interface of CT and ML, there is a risk that some of the expected outcomes will not be achieved. However, as the Action has sufficiently diversified the proposed research, such unfortunate events should not significantly affect the overall success of the project. In case that some tasks exhibit unexpected challenges in bringing together the theoretical results from CT and ML, there is always the possibility to address computational and application-oriented tasks first; the respective WG leaders will then adjust the research agenda appropriately. The latter tasks not only provide progress in their own right, but also stimulate intuition and induce theoretical research questions, e.g., for analysing the reliability and improving the performance of numerical algorithms. This, in turn, will provide an additional impulse to theoretical investigations and eventually allow for novel insights and results.
- **Imbalance of expertise.** As the Action proposes research at the intersection of many fields, the initial group is multidisciplinary, with an excellent balance between theory, applications, and numerical methods. However, it might happen that more critical mass in some areas will be required. In this case, suitable participants will be identified and approached by the Core Group, in order to widen the network and improve the balance of expertises.
- **Gender, age, geographical or other imbalances.** Although the initial network of proposers was fairly balanced, there exists a risk that this will not be the case for certain Action activities or that the overall balance will be jeopardised as the Action (organically) expands. To avoid this risk, age, gender and geographical factors will be included as selection criteria for participation in the Action and its events, as well as for evaluating applications for grants. The situation will be continuously monitored and the EOC will prepare annual progress reports, including the gender, geographic and age distribution of participants in the Action's activities, proposing solutions to improve the balance if necessary.
- **Low participation in Training Schools & STSMs.** In order to minimise this risk the Action will promote the events through its own communication channels, but also through social media, mailing lists of scientific associations, and personal communication. Feedback from participants will be regularly collected and will be taken into account in the organisation of subsequent events.
- **Lack of communication.** Due to the large number of participants, communication and

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