List of open problems related to Working Group 2 – ML for CT

December 2025

T2.1 Addressing the curse of dimensionality with ML tools

1. **Learn set-valued maps related to control problems with machine learning tools**
   - *Contact:* Francisco Periago. Email: f.periago@upct.es
   - *Required skills:* Good command of Python and a basic knowledge of control theory
2. **Regularity theory for PDEs in high dimensions**
   - *Contact:* Francisco Periago. Email: f.periago@upct.es
   - *Required skills:* Good command of functional analysis and PDEs

T2.2 Solving parameterised optimal control problems

1. **Nonlinear and transport-dominated problems**
   - *Goal:* Solve optimal control problems where the governing dynamics are parametric and nonlinear or transport-dominated, for instance

$$\frac{\partial y}{\partial t} + \mu \frac{\partial y^2}{\partial x} = u.$$

   - *Contact:* Martin Lazar. Email: mlazar@unidu.hr
   - *Required skills:* Good command in numerics of PDEs and in control theory
   - *Some details/related questions:*
     - Nonlinear problems pose difficulties for traditional, linear approximation schemes.
     - General question for model order reduction.
     - What is possible in the context of (optimal) control of such systems?
     - Where can machine learning help to overcome these issues (nonlinear strategies such as autoencoders, etc.)?
2. **Control of flow problems such as the Navier-Stokes equations**
   - *Goal:* Solve optimal control problems for flows governed by parametric Navier-Stokes equations, for instance

$$\begin{aligned} \frac{\partial y}{\partial t} - \mu \Delta y + (y \cdot \nabla) y + \nabla p &= u, \\ \operatorname{div} y &= 0. \end{aligned}$$

   - *Contact:* Maria Strazzullo. Email: maria.strazzullo@polito.it
   - *Required skills:* Good command in numerics of PDEs and in control theory
   - *Some details/related questions:*
     - Navier-Stokes equations are an important model for (viscous) flow in real-world applications.

- Depending on the Reynolds number (roughly the parameter μ in the formulation above), the solution behavior can change completely.
- Can machine learning help in order to deal with the turbulent regime?

3. **General convex objective functionals**

   - *Goal:* Induce sparsity in the control by solving a problem of the form

   $$u_\mu^* = \arg\min_{u \in G} \|u\|_{L^1([0,T];U)} + \frac{\alpha}{2}\|u\|_{L^2([0,T];U)}^2 + h(x_\mu)$$

   - *Contact:* Cesare Molinari. Email: cesare.molinari@edu.unige.it
   - *Required skills:* Good command in (convex) optimization; Python programming
   - *Some details/related questions:*
     - Which algorithm is suited best to solve this OCP?
     - How to deal with the parameter dependence?
     - Can reduced order modeling be applied here in a suitable manner?
     - If so, how to combine it in a reasonable way with machine learning?

4. **Optimization in the parameter space**

   - *Goal:* Solve problems of the form

   $$\mu^* = \arg\min_{\mu \in \mathcal{P}} F(\mu; u_\mu)$$

   where $u_\mu \in G$ solves an optimal control problem for the parameter $\mu \in \mathcal{P}$.
   - *Contact:* Hendrik Kleikamp. Email: hendrik.kleikamp@uni-graz.at
   - *Required skills:* Knowledge in optimization and control theory; Python programming
   - *Some details/related questions:*
     - Optimal control problem for a fixed parameter as an "inner" problem.
     - Derivatives with respect to the parameter are typically required.
     - Optimizer usually moves outside ot the range of training data points.
       $\longrightarrow$ How to extrapolate properly?

5. **Small data regime**

   - *Goal:* How to deal with relatively small amount of available data?
   - *Contact:* Hendrik Kleikamp. Email: hendrik.kleikamp@uni-graz.at
   - *Required skills:* Good command of machine learning and control theory
   - *Some details/related questions:*
     - Training data (at least using the FOM) is costly to obtain.
     - Which quantities are easiest to learn when only a small amount of data is available?
       * Optimal control $\longrightarrow$ How to obtain performance guarantees?
       * Reduced quantities $\longrightarrow$ Combination with MOR techniques often allows to collect more training data and to make use of their error estimates.
       * Open loop vs. closed loop systems $\longrightarrow$ Feedback control requires different architectures and learning techniques.

6. **Applications in uncertainty quantification**

   - *Goal:* Make use of the derived surrogates in *multilevel Monte Carlo methods*:

   $$\mathbb{E}[M_L] = \mathbb{E}[M_0] + \sum_{\ell=0}^{L} \mathbb{E}[M_\ell - M_{\ell-1}].$$

- *Contact:* Hendrik Kleikamp. Email: hendrik.kleikamp@uni-graz.at
- *Required skills:* Machine learning and surrogate modeling; a bit of probability theory and statistics; Python programming
- *Some details/related questions:*
  - Consider different applications in which we want efficient estimates of unknown quantities.
  - Interactions of the different models?
  - Strategies to select the models and the number of evaluations on different levels?
  - Can we derive probabilistic guarantees that this works?

T2.3 Construction of control Lyapunov functions using ML methods

1. **Fast and reliable learning algorithms (in particular for nonsmooth functions)**
   - *Contact:* Lars Grüne. Email: lars.gruene@uni-bayreuth.de
2. **Efficient verification of a control Lyapunov function candidate**
   - *Contact:* Lars Grüne. Email: lars.gruene@uni-bayreuth.de

T2.4 Developing ML-based approaches for the life-cycle-optimisation in materials

1. **Approximating solutions to the elasticity system**
   - *Goal:* Develop the concept of approximating solutions to the elasticity system with damage evolution.
   - *Contact:* Peter Kogut. Email: p.kogut@i.ua
2. **Existence and uniqueness of weak solutions I**
   - *Goal:* Establish the existence and uniqueness of the weak solutions via approximation for the L^1-damage source function
   $$\phi(\mathbf{e}(\mathbf{u}),\zeta) = -\lambda_D\left(\frac{1-\zeta}{\zeta}\right) - \frac{1}{2}\lambda_u \mathbf{e}(\mathbf{u})\cdot\mathbf{e}(\mathbf{u}) + \lambda_w.$$
   - *Contact:* Peter Kogut. Email: p.kogut@i.ua
3. **Existence and uniqueness of weak solutions II**
   - *Goal:* Study the existence of weak solutions to the original problem using the following relaxed version
   $$-\operatorname{div}(\zeta A\mathbf{e}(\mathbf{u})) + \varepsilon\mathbf{u} = \mathbf{f} \qquad \text{in } \Omega_T.$$
   - *Contact:* Peter Kogut. Email: p.kogut@i.ua
4. **Investigating the strain tensor**
   - *Goal:* Find out whether the strain tensor $\mathbf{e}(\mathbf{u}) = \{\mathbf{e}_{ij}/\mathbf{u}\}$ with
   $$\mathbf{e}_{ij}(\mathbf{u}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \qquad \forall i,j = 1,\dots,N$$
   possesses the high integrability property, $|\mathbf{e}(\mathbf{u})| \in L^{2(1+\delta)}$ for some $\delta > 0$.
   - *Contact:* Peter Kogut. Email: p.kogut@i.ua
5. **Existence of an optimal control**

- *Goal:* Establish the existence of an optimal control provided the damage source function takes the form

$$\phi(\mathbf{e}(\mathbf{u}), \zeta) = -\lambda_D \left(\frac{1-\zeta}{\zeta} \right) - \frac{1}{2} \lambda_u \mathbf{e}(\mathbf{u}) \cdot \mathbf{e}(\mathbf{u}) + \lambda_w.$$

- *Contact:* Peter Kogut. Email: p.kogut@i.ua

6. **Controls in the limit of vanishing smoothing**

- *Goal:* Find out whether sustainable controls can be attained in the limit as $\varepsilon \to 0$ using the following relaxed version of the first equation

$$-\operatorname{div}((\zeta)_\varepsilon A \mathbf{e}(\mathbf{u})) = \mathbf{f} \qquad \text{in } \Omega_T,$$

where $(\cdot)_\varepsilon$ stands for the Steklov smoothing operator.

- *Contact:* Peter Kogut. Email: p.kogut@i.ua

7. **Existence of a control**

- *Goal:* It is unknown whether there exists a control $f \in \mathcal{F}_{ad}$ such that the corresponding solutions $(\zeta, \mathbf{u})$ satisfy the equations

$$\begin{aligned} -\operatorname{div}(\zeta A \mathbf{e}(\mathbf{u})) &= \mathbf{f}, \\ \zeta' - \kappa \Delta \zeta &= \phi(\mathbf{e}(\mathbf{u}), \zeta) \end{aligned}$$

in the sense of $L^2(Q_T)$.

- *Contact:* Peter Kogut. Email: p.kogut@i.ua

T2.5 Exploiting PINNs for solving complex free boundary problems